Zadání zápočtových a zkouškových písemek z limit v kurzu Diferenciální počet na PedF UK z let 2005/6 a 2006/7:

A. Bez užití L'Hospitalova pravidla a Taylorových polynomů spočtěte limitu:

1. $\lim\limits_{x\to 2} \dfrac{\sqrt{x^2-4x+7}-\sqrt{9-3x}}{2x^3-9x^2+12x-4}$
2. $\lim\limits_{x\to -\infty} \left(\sqrt{4x^2-3}+2x\right)\ln\left(x^4+2^x\right)$
3. $\lim\limits_{x\to -3} \dfrac{x^3-7x+6}{\left(\sqrt{x^2+6x+10}-\sqrt{2x+7}\right)^2}$
4. $\lim\limits_{x\to -\infty} \dfrac{1-\cos\frac{1}{x}}{\left(3x+\sqrt{9x^2-2}\right)^2}$
5. $\lim\limits_{x\to \infty} \dfrac{\ln\left(\frac{2}{\pi}\operatorname{arctg} x\right)}{\sqrt{4x^2+4x+1}-2\sqrt{x^2+x}}$
6. $\lim\limits_{x\to 0+} \mathrm{e}^{x-\ln x}\left(\cos\sqrt{3^x-1}-1\right)$
7. $\lim\limits_{x\to \infty} \ln\left(\dfrac{2x^2-x+1}{2x^2+x-1}\right)(x+\sin x)$
8. $\lim\limits_{x\to 2} \dfrac{\ln(x^2-3)}{x^6-12x^4+48x^2-64}$
9. $\lim\limits_{x\to \infty} \dfrac{\operatorname{arccotg}\frac{x^4}{1+x^2}}{\cos\frac{x}{x^2+1}-1}$
10. $\lim\limits_{x\to 0} (\cos x)^{\operatorname{arccotg}^{-1} x^{-2}}$
11. $\lim\limits_{x\to 1} \dfrac{3}{x^3-1}-\dfrac{5}{x^5-1}$
12. $\lim\limits_{x\to 2} \dfrac{\ln\frac{4^x}{2^{x^2}}}{x^3-2x^2+x-2}$
13. $\lim\limits_{x\to \infty} \ln^2 x^2-\ln^2(x^2+1)$
14. $\lim\limits_{x\to \infty} \dfrac{\ln x-\ln\sqrt{x^2+1}}{\operatorname{arccotg} x}$
15. $\lim\limits_{x\to 0} \dfrac{\ln\left(\frac{1+\cos x}{2}\right)}{\mathrm{e}^{x^2}-1}$
16. $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+1}-1}{\operatorname{tg} x-\sin x}$
17. $\lim\limits_{x\to -1} \dfrac{1-\sqrt{3+x-x^2}}{\ln(x^5+10x^2+15x+7)}$
18. $\lim\limits_{x\to 0} \dfrac{\arccos \mathrm{e}^{-x^2}}{x}$
19. $\lim\limits_{x\to 0} \dfrac{\ln(\cos x+\sin x)+\ln(\cos x-\sin x)}{\sqrt{x^3+2x^2+2x+1}-x-1}$
20. $\lim\limits_{x\to 0} \dfrac{2\sin x-\sin 2x}{\operatorname{tg}(\sin(\operatorname{arctg} x^3))}$
21. $\lim\limits_{x\to 0} \dfrac{\ln(\sin x-\cos x)^2}{\sqrt{1-\cos x}}$
22. $\lim\limits_{x\to \infty} \left(\sqrt{x^2+2x}-x\right)^x$
23. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1+\operatorname{tg} x}}{\operatorname{arctg} x^3}$
24. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin^2 x}-\cos x}{\ln\cos x}$
25. $\lim\limits_{x\to 2} \dfrac{\sqrt{1-\cos(\pi x)}}{\sin(3x^3-10x^2+4x+8)}$
26. $\lim\limits_{x\to \infty} (\mathrm{e}^{-x}+\operatorname{arccotg} x)\ln(x^{\sqrt{x}}+x^2)$
27. $\lim\limits_{x\to 1} \dfrac{\ln x}{\arccos\frac{2x}{x^2+1}}$
28. $\lim\limits_{x\to -1} \dfrac{2^{-x}-2}{\ln(x-3\sqrt[3]{x}-1)}$
29. $\lim\limits_{x\to -\infty} x\sin\dfrac{\operatorname{arctg}^2\frac{1}{x}}{\operatorname{arccotg} x-\pi}$
30. $\lim\limits_{x\to 0} (\cos x)^{(1-\sqrt{1-\sin x^2})^{-1}}$
31. $\lim\limits_{x\to \infty} x\arccos\dfrac{2\operatorname{arctg} x^2}{\pi}$
32. $\lim\limits_{x\to 1} \dfrac{2^x-2\sqrt{2^x-1}}{\ln(2x^5-5x^4+10x^2-10x+4)}$
33. $\lim\limits_{x\to 1} \dfrac{\sqrt{\ln^2 x+1}-\ln x-1}{\sin(2\pi x)}$

B. Bez užití L'Hospitalova pravidla a Taylorových polynomů spočtěte limity funkce f v krajních bodech $\mathcal{D}(f)$:

1. $f(x)=\dfrac{\sqrt{x^2+1}}{x\operatorname{arctg}(2^x-1)}$
2. $f(x)=\dfrac{\sqrt{\ln(1+x)}}{\ln(1+\sqrt{x})}$
3. $f(x)=\dfrac{\sin \mathrm{e}^x}{\mathrm{e}^x-\mathrm{e}^{2x}}$
4. $f(x)=\dfrac{\ln(\mathrm{e}^x-x)-x}{\operatorname{arctg} x}$
5. $f(x)=\dfrac{x\mathrm{e}^x}{\sqrt{\mathrm{e}^{x^2}-1}}$
6. $f(x)=\dfrac{3^x\sin\frac{1}{x}}{\mathrm{e}^x-\ln x}$
7. $f(x)=\dfrac{\operatorname{arctg}(x\,\mathrm{e}^{-x})(\mathrm{e}^x+1)}{\sqrt{x^2-x+1}-x-1}$
8. $f(x)=\dfrac{\sin\frac{x^4}{x^2+1}}{\ln\frac{x^4+x^2+1}{x^2+1}}$

9. $f(x) = \dfrac{\sqrt{x^2+1}+x-1}{\mathrm{e}^{2x}+\mathrm{e}^x-2}$

10. $f(x) = \dfrac{\mathrm{e}^x \sin x}{x^x-1}$

11. $f(x) = \dfrac{\operatorname{arctg} x}{(x+\sin x)\operatorname{arccotg} x}$

12. $f(x) = \dfrac{\operatorname{arctg} x^2}{\left(\sqrt{x^2+2x+4}-x-2\right)^2}$

13. $f(x) = \dfrac{\sin\left(x+\sqrt{x^2+1}\right)}{\mathrm{e}^x}$

14. $f(x) = \dfrac{\operatorname{arctg} x \operatorname{arccotg} x}{\sqrt{x^2+x+1}-x-1}$

15. $f(x) = \mathrm{e}^x \sin\left(x \operatorname{arccotg}^2 x\right)$

16. $f(x) = \dfrac{\sin\left((\pi-\operatorname{arccotg} x)\sqrt[3]{x}\right)}{\mathrm{e}^{\mathrm{e}^x}-1}$

17. $f(x) = \dfrac{\sin\left(x^2\mathrm{e}^{-x}\right)}{1-\sqrt{x^2+1}}$

18. $f(x) = \operatorname{arccotg}(-x) \cdot \sin\dfrac{\sqrt{x^2+1}-1}{\left(\mathrm{e}^x-2^x\right)^2}$

19. $f(x) = \dfrac{1-\cos\sqrt{x}}{\ln\frac{x+1}{x^2+1}}$

20. $f(x) = \dfrac{\cos \mathrm{e}^x-1}{3^x-2^x}$

21. $f(x) = \dfrac{\mathrm{e}^{\sqrt{x^2+1}-x}-1}{(\mathrm{e}^{-x}+1)\operatorname{arccotg} x}$

22. $f(x) = \dfrac{\sin(\sqrt{x^2+2x}-x-1)}{\operatorname{arccotg} x}$

23. $f(x) = \dfrac{\sqrt{2-\cos x}-1}{\mathrm{e}^{x^2}-1}$

24. $f(x) = \dfrac{\sin(\cos x-1)}{\ln\frac{x^4+1}{x^2+1}}$

25. $f(x) = \dfrac{x}{(\sqrt{x^2+x+1}+x)^2}\ln\dfrac{\operatorname{arccotg} x}{\pi}$

26. $f(x) = \dfrac{\sin(x \operatorname{arccotg} x)(\sqrt{x^2+x+1}+x)}{\operatorname{arctg} x}$

27. $f(x) = \dfrac{(\sqrt{\ln^2 x+x}+\ln x)\ln x}{x}$

28. $f(x) = \sin(x \operatorname{arccotg} x^2)\sqrt{x^2+\dfrac{1}{x^2}}$

29. $f(x) = \dfrac{\sin\ln\frac{x^2}{x^2+1}}{(\mathrm{e}^{\frac{1}{x}}-1)^2}$