

Nalezněte hodnotu limity a dokažte z definice:

$$\begin{array}{llll}
 1. \lim \frac{2n-3}{3n-2} & 2. \lim \frac{2-n}{2n+2} & 3. \lim \frac{(-1)^n n}{n^2+1} & 4. \lim \frac{n^2-4n}{n^2-4} & 5. \lim \frac{n^2+n}{n^2-2} \\
 6. \lim \frac{n^2-n}{n^2-n-2} & 7. \lim \frac{n^2-n}{n+2} & 8. \lim \frac{(n-2)^2}{1-n} & 9. \lim \frac{\sin n}{n} &
 \end{array}$$

Nalezněte hodnotu limity posloupnosti (a_n) a dokažte z definice. Dále zjistěte, pro která n je a_n větší (resp. menší, resp. rovno) a_{n+1} , určete supremum, infimum, maximum a minimum (a_n) a rozhodněte, zda je omezená.

$$\begin{array}{ll}
 10. a_n = \frac{1}{n^2-15n+18}, n \in \mathbb{N}_0 & 11. a_n = \frac{n-4}{n^2-13n+28}, n \in \mathbb{N}_0 \\
 12. a_n = \frac{n-13}{n^2-11n-3}, n \in \mathbb{N}_0 & 13. a_n = \frac{n^2-12n+38}{2n-17}, n \in \mathbb{N}_0
 \end{array}$$

Spočítejte limity nebo dokažte, že neexistují:

$$\begin{array}{ll}
 14. \lim \frac{n^3+n^2+n+1}{n^4} & 15. \lim \frac{(2n-1)^3}{(1-n)(1+3n)} \\
 16. \lim \frac{(3n^2+2n+1)^4(1-n)^6}{((1+n)^3+(1-n)^3)^7} & 17. \lim \frac{\sqrt{n^2+1}}{n+1} \\
 18. \lim \sqrt{2n+3} - \sqrt{n} & 19. \lim \sqrt{n+3} - \sqrt{n} \\
 20. \lim \sqrt{n^2+3n} - n & 21. \lim \sqrt{n+3} - n \\
 22. \lim \sqrt{n+\sqrt{n}} - \sqrt{n-\sqrt{n}} & 23. \lim \sqrt{1+n^2} + \sqrt{n-n^2} \\
 24. \lim n \left(\sqrt{n^2+2n} - n - 1 \right) & 25. \lim (\sqrt{n+2} - \sqrt{3n-1}) (\sqrt{4n+3} - 2\sqrt{n-1}) \\
 26. \lim \frac{(\sqrt{n^4+n^2+1} - n\sqrt{n^2+n+1})^2}{\sqrt[3]{n^2+n} - \sqrt{n^3-1}} & 27. \lim \frac{\sqrt{n+\sqrt{n^2-n}} - \sqrt{n+\sqrt{n^2+1}}}{\sqrt{n+3} - \sqrt{n+1}} \\
 28. \lim \sqrt[4]{n^4+3n^3} - \sqrt[4]{n^4-1} & 29. \lim \sqrt[3]{1+n^3} - \sqrt[3]{n-n^3} \\
 30. \lim \sqrt[3]{1+n^3} + \sqrt[3]{n-n^3} & 31. \lim \sqrt{n^2+2} - \sqrt[3]{n^3+3n} \\
 32. \lim 3\sqrt{n^2+n} + \sqrt{n^2-1} - 4n & 33. \lim n^2(\sqrt{n+1} - 2\sqrt{n} + \sqrt{n-1}) \\
 34. * \lim n^2(\sqrt{n+3} - 3\sqrt{n+2} + 3\sqrt{n+1} - \sqrt{n}) & 35. \lim \sin^n 1 \\
 36. \lim \frac{3^n 2^n}{4^n} & 37. \lim (-1)^n \frac{7 \cdot 3^{n+3}}{2^{4n-1} 5^{1-n}} \\
 38. \lim \frac{4^n - 5^n}{3^n + 2^n} & 39. \lim (-1)^n \frac{4^n + 5^{1-n}}{3^{n+1} - 2^{2n}} \\
 40. \lim (-1)^n \frac{(-4)^n + (-5)^{1-n}}{(-3)^{n+1} - (-2)^{2n}} & 41. \lim \frac{5 \cdot 4^{n+2} \cdot 9^{n+1} - 3 \cdot 6^{2n+3}}{(3n-7)^2(2^n-2^{-n})^2}
 \end{array}$$

42. $\lim (3^{\frac{n}{2}} - 2^{n-1}) (3^{-\frac{n}{2}} - 2^{1-n})$ 43. $\lim \sqrt[n]{\sqrt{2^n} + \sqrt[3]{3^n}}$ 44. $\lim \sqrt[n]{\sqrt[5]{5^n} - \sqrt[4]{4^n}}$
45. $\lim 2^{n+1} \sqrt{2^{2n} - 5^n}$ 46. $\lim \left(\frac{n}{n+1}\right)^n$ 47. $\lim \left(\frac{n}{n-1}\right)^{n+1}$
48. $\lim \left(\frac{2n-2}{2n+1}\right)^{3n+1}$ 49. $\lim \left(\frac{3n+1}{2n+3}\right)^{2n+3}$ 50. $\lim \left(\frac{2n-3}{2n-1}\right)^{n^2}$
51. $\lim \left(\frac{2n^2-n+1}{2n^2-n-3}\right)^{n-2}$ 52. $\lim \left(\frac{1-n}{2-n}\right)^{3-n}$ 53. $\lim 2^{2n+1} \left(\frac{n}{4n+1}\right)^n$
54. $\lim \left(\frac{1-n}{1+n}\right)^{2n+1}$ 55. $\lim \left(\frac{1-n}{1+n}\right)^n$ 56. $\lim \left(\frac{1-n}{1+n}\right)^{n^2}$
57. $\lim \left(\frac{\ln en}{\ln n}\right)^n$ 58. $\lim \frac{n^n(n+1)^{n+1}}{(n-1)^{n-1}(n+2)^{n+2}}$ 59. $\lim \frac{n^2}{2^n}$
60. $\lim \frac{n!}{2^n}$ 61. $\lim \frac{2^{n^2}}{8^n}$ 62. $\lim \frac{(2n)!(3n)!}{n!(4n)!}$
63. $\lim \frac{2^{2^n}}{n!^n}$ 64. $\lim \frac{\ln n}{n}$ 65. $\lim \frac{\ln n}{\ln \ln n}$
66. $\lim \frac{\ln^3 n}{\sqrt{n}}$ 67. $\lim \frac{n!}{4!^n}$ 68. $\lim \frac{2^n + n^2}{\ln n - n^n}$
69. $\lim \frac{2^n n^2 \ln n}{n!}$ 70. $\lim \frac{\ln(3n^2)}{\ln n}$ 71. $\lim \frac{\ln(2^n + n)}{\ln n}$
72. $\lim \frac{\ln(2^{-n} + n)}{\ln(2^{-n} + n^{-1})}$ 73. $\lim \sqrt[n]{2}$ 74. $\lim 2^{n-1} \sqrt{-2}$
75. $\lim 3^{n-1} \sqrt{-3}$ 76. $\lim \sqrt[n]{n}$ 77. $\lim \sqrt[n]{n!}$
78. $\lim 4^{n+1} \sqrt[n]{n!}$ 79. $\lim \sqrt[n^2]{n!}$ 80. $\lim \sqrt[n]{n!}$
81. $\lim (-1)^n n$ 82. $\lim \frac{(-1)^n \sqrt{n}}{n+1}$ 83. $\lim \frac{(-1)^n n}{(-1)^n + n}$
84. $\lim (-1)^{n^2} \cos \pi n$ 85. $\lim \sin(2^n \pi)$ 86. $\lim \sin \frac{n\pi}{4}$
87. $\lim \sin^2 \frac{(2n+1)\pi}{4}$ 88. $\lim \sin n$ 89. $\lim \operatorname{tg} n$
90. $\lim \operatorname{tg} \frac{17\pi n}{26}$ 91. $\lim \frac{\sin n^2}{n^2}$ 92. $\lim \frac{\sin 2^n - n}{n - \ln n}$
93. $\lim \frac{n^2 \sin n! - 3^n \ln n}{\pi^n}$ 94. $\lim \sqrt{\frac{2-n}{n^2+1}}$ 95. $\lim \arcsin \frac{n+1}{n-1}$
96. $\lim \left(\frac{1}{n^2} + \frac{3}{n^2} + \frac{5}{n^2} + \dots + \frac{2n-1}{n^2}\right)$ 97. $\lim \left(\frac{n}{n^2} + \frac{n+1}{n^2} + \frac{n+2}{n^2} + \dots + \frac{2n}{n^2}\right)$
98. $\lim \left(\frac{1}{\sqrt{8}} + \frac{2}{\sqrt{35}} + \dots + \frac{n}{\sqrt{9n^2-1}}\right)$ 99. $\lim (1 - 2 + 3 - 4 + \dots - 2n)$
100. $\lim (1 - 2 + 3 - 4 + \dots + (-1)^{n-1} n)$

Určete limity v závislosti na parametru či parametrech:

$$101. \lim \frac{\sqrt{n^3 - n^2} - \sqrt{n^3 + 1}}{n^k}, \quad k \in \mathbb{R}$$

$$102. \lim \sqrt{n^2 - an + 1} - \sqrt{bn^2 + n}, \quad a, b \in \mathbb{R}$$

$$103. \lim \frac{an^2 + b}{cn + 1}, \quad a, b, c \in \mathbb{R}$$

$$104. \lim (-1)^n (x^2 - 3x)(x - 1)^{n+1} (3 - x)^{n+3}, \quad x \in \mathbb{R}$$

$$105. \lim (x + 4)(x + 9)^{n+1} (2x + 6)^{1-n}, \quad x \in \mathbb{R}$$

$$106. \lim 4^{\frac{n-1}{2}} \left(\cos \frac{x}{2} + \frac{1}{2} \right) \left(\sin x - \frac{1}{2} \right)^{n+1}, \quad x \in \mathbb{R}$$